A coordinating contract for transshipment in a two-company supply chain

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Introduction

◆ The transshipment provides a company with the option to buy another company’s surplus, if any, whenever it faces unsatisfied demand.

◆ An example of this practice is discernible in the oil industry where volatility of demands and limitation of regional refinery capacities make transshipment a reasonable practice (Dempster et al., 2000).

◆ We use the Generalized Nash Bargaining Solution (GNBS) (Roth, 1979) to find a coordinating contract for the transshipment problem.
Rudi et al. (2001) study a single-period transshipment problem with two independent retailers. They derive the transshipment prices that cause the independent retailers to choose the first best production quantities.

However, Hu et al. (2007) prove that such transshipment prices may exist only under certain conditions, thus not always.

Moreover, even such coordinating transshipment prices exist, they can’t allow for dividing the supply chain profits arbitrarily.

Hence, We propose a coordinating contract with an implicit pricing mechanism that can coordinate the two companies.
The model

Assumption:

- We consider a system with two risk-neutral companies \((i, j = 1, 2)\) producing an identical product.

- \(Q = (Q_1, Q_2)\) production quantities of the companies, prior to \(D\)

- \(D = (D_1, D_2)\) random demands, bivariate continuous and twice differentiable

- \(c = (c_1, c_2)\) production costs of one unit

- \(r = (r_1, r_2)\) selling prices of one unit

- \(s = (s_1, s_2)\) salvage values of one unit

subject to: \(0 \leq s < c < r\)

- \(b = (b_1, b_2)\) penalty of each unsatisfied demand

- \(p = (p_{1i}, p_{2i})\) where \(p_{ij}\) is the unit price that \(i\) should pay in order to receive a unit of \(j\)'s surplus product

- \(\tau = (\tau_{ij}, \tau_{2i})\) when \(i\) transships to \(j\), the former incurs a unit transportation cost
The model

To assure that the transshipment occurs only if one company has unsatisfied demands and the other has surplus, it is commonly assumed (see Rudi et al. (2001) for example) that

\[ c_i < c_j + \tau_{ji} \quad s_i < s_j + \tau_{ji} \quad r_i + b_i < r_j + b_j + \tau_{ji} \quad \text{for} \quad i, j = 1, 2 \]

To make sure the transshipment is feasible if neither company is worse off by doing it. Therefore, the transshipment prices will be feasible if

\[ \tau_{ij} + s_i \leq p_{ij} \leq r_j + b_j \quad \text{for} \quad i, j = 1, 2 \] (1)
The process is as following:

- The variables: $Q_1, Q_2, p_{12}, p_{21}$

In the diagram:

- $c_1, s_1, b_1$ for Company 1
- $c_2, s_2, b_2$ for Company 2
- $B(Q)$
- $A(Q)$
- $\tau_{12}$
- $\tau_{21}$
- $r_1$ from $Q_1$ to $D_1$
- $r_2$ from $Q_2$ to $D_2$
The model

- We use the superscripts \( n \) and \( d \) to distinguish between non-cooperative and cooperative modes respectively.

- **Non-cooperative mode (without transshipment):**

  \[
  \pi_i^n(Q_i) = E\{r_i \min(D_i, Q_i) + s_i(Q_i - D_i)^+ - b_i(D_i - Q_i)^+\} - c_i Q_i, \tag{2}
  \]

  It’s just a newsvendor problem, and we denote the total expected profit in this mode by \( \pi^n(Q) = \pi^n(Q_1) + \pi^n(Q_2) \).

- **Cooperative mode:**

  \[
  (T_{ij}(Q) = \min([D_j - Q_j]^+, [Q_i - D_i]^+)) \text{: the transshipment amount from } i \text{ to } j)
  \]

  \[
  \begin{align*}
  \pi_1^c(p, Q) &= (p_{12} - \tau_{12} - s_1)A(Q) + (r_1 + b_1 - p_{21})B(Q) + \pi_1^n(Q_1), \\
  \pi_2^c(p, Q) &= (r_2 + b_2 - p_{12})A(Q) + (p_{21} - \tau_{21} - s_2)B(Q) + \pi_2^n(Q_2),
  \end{align*} \tag{3}
  \]

  \[
  \pi_1^c(Q) = (r_2 + b_2 - \tau_{12} - s_1)A(Q) + (r_1 + b_1 - \tau_{21} - s_2)B(Q) + \pi_1^n(Q_1) + \pi_2^n(Q_2), \tag{4}
  \]

  so,

  \[
  \pi_1^c(Q) = (r_2 + b_2 - \tau_{12} - s_1)A(Q) + (r_1 + b_1 - \tau_{21} - s_2)B(Q) + \pi_1^n(Q_1) + \pi_2^n(Q_2),
  \]

  where

  \[
  A(Q) = E[T_{12}(Q)] \text{ and } B(Q) = E[T_{21}(Q)].
  \]

  Then, we can solve the optimum production quantities, just as the first best quantities \( Q^c = (Q_1^c, Q_2^c) \).
Using the concept of Generalized Nash Bargaining Solution, consider two players (here 1 and 2). Assuming $\alpha \in (0,1)$ is the player 1’s bargaining power over player 2. Then, the bargaining solution can be formulated as:

$$f_2 = \arg \max_p \left[ \pi_1^1(p, Q) - \pi_2^1(Q^1) \right]^x \left[ \pi_2^2(p, Q) - \pi_2^2(Q^2) \right]^{1-x}.$$ \hspace{1cm} (6)

**Lemma 1 (The GNBS condition).** For any $Q$, the transshipment prices which solve (6), $p^* = (p^*_{12}, p^*_{21})$, satisfy the following condition:

$$A(Q)p_{12} - B(Q)p_{21} = \left[ \alpha(r_2 + b_2) + (1 - \alpha)(\tau_{12} + s_1) \right]A(Q) - \left[ (1 - \alpha)(r_1 + b_1) + \alpha(\tau_{21} + s_2) \right]B(Q)
+ \alpha \left[ \pi_2^2(Q_2) - \pi_2^2(Q^2_2) \right] - (1 - \alpha) \left[ \pi_1^1(Q_1) - \pi_1^1(Q^1_1) \right].$$ \hspace{1cm} (7)

**Three situations:**

1. Both $A(Q) \neq 0$ and $B(Q) \neq 0$, the companies will have several alternatives for fixing $p^*(Q)$.
2. Either $A(Q) = 0$ or $B(Q) = 0$ (but not both), there’ll be only one choice.
3. The case with $A(Q) = B(Q) = 0$, no transshipment exists, just as the non-cooperative situation.

$p^*(Q)$ is an implicit pricing mechanism.
The contract

- The two rounds of the contract are showed in the following figure:

- Then, we try to decide the production quantities. By inputting eq (7) into eq(3) and (4), we get lemma 2:

  Lemma 2. With \( p^*(Q) \), the expected individual profits are

  \[
  \pi_1^i(p^*(Q), Q) = \alpha \pi_1^i(Q) + [(1 - \alpha)\pi_1^i(Q_1^i) - \alpha \pi_2^i(Q_2^i)], \\
  \pi_2^i(p^*(Q), Q) = (1 - \alpha)\pi_2^i(Q) - [(1 - \alpha)\pi_1^i(Q_1^i) - \alpha \pi_2^i(Q_2^i)].
  \]

- Lemma 2 states that with \( p^*(Q) \), the expected individual profit for each company equals its maximum expected profit in the non-cooperative mode, plus a fraction (\( a \) for company 1 and \( 1 - a \) for company 2) of expected extra profit resulting from the cooperation.
Then we have theorem,

**Theorem 1.** With \( p'(Q) \), \( Q^d = Q^c \).

The second round of the contract—fixing the negotiated transshipment prices

Let \( \Omega(Q) \) be the set of all transshipment prices for a given \( Q \). Then, considering eq(1), eq(7), we get

\[
\Omega(Q) = \begin{cases} 
\max(L_1, \tau_{21} + s_2) \leq p^*_2 \leq \min(L_2, r_1 + b_1) & \text{if } A(Q) \neq 0 \text{ and } B(Q) \neq 0, \\
\tau_{21} + s_2 \leq p^*_2 \leq r_1 + b_1 & \text{if } A(Q) = 0 \text{ and } B(Q) \neq 0, \\
\tau_{12} + s_1 \leq p^*_1 \leq r_2 + b_2 & \text{if } A(Q) \neq 0 \text{ and } B(Q) = 0,
\end{cases}
\]  

where

\[
L_1 = -\alpha(r_2 + b_2 - \tau_{12} - s_1) A(Q) B(Q) - \frac{\alpha(\pi^2(Q_2) - \pi^2_2(Q_2^c)) - (1 - \alpha)[\pi^1_1(Q_1) - \pi^1(Q_1^c)]}{B(Q)} + (1 - \alpha)(r_1 + b_1) + \alpha(\tau_{21} + s_2),
\]

\[
L_2 = (1 - \alpha)(r_2 + b_2 - \tau_{12} - s_1) A(Q) B(Q) - \frac{\alpha(\pi^2(Q_2) - \pi^2_2(Q_2^c)) - (1 - \alpha)[\pi^1_1(Q_1) - \pi^1(Q_1^c)]}{B(Q)} + (1 - \alpha)(r_1 + b_1) + \alpha(\tau_{21} + s_2).
\]  

**Theorem 2.** \( \Omega(Q^c) \) is non-empty.

,so we know the coordinating transshipment prices can always exists.
Simulation

◆ Using the example proposed in Hu et al. (2007) as an instance where there is no linear transshipment prices that results in the first best production quantities.

◆ By giving specific values, considering simple demand, we simulate the profit functions in three situations.

◆ Through simulation, we show just implicit pricing mechanism can leads to the coordination of this system.
Conclusion & future work

◆ We proposed a contract with an implicit pricing mechanism that can coordinate the transshipments in a two-company supply chain.

◆ This contract has several desirable properties.
  • First, the implicit pricing mechanism gives rise to the choice of first best production quantities.
  • Second, the implicit pricing mechanism allows for arbitrary division of total expected extra profit according to the bargaining powers.
  • Third, when the companies fix the negotiated transshipment prices they usually have multiple alternatives to chose from.
Some directions for future work

- Extend the single echelon to more echelons, such as a supplier and two retailers, with transshipment in the two retailers.
- Consider the efficiencies of different contracts and the contracting cost for different transshipment pricing mechanisms, when choosing contracts.
- Consider two dependent companies, such as including the competition among companies when choosing their market selling prices.
Thank You!

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